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SINGH'S FUZZY TIME SERIES *FORECASTING MODIFICATION* BASED ON INTERVAL RATIO

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Keywords:
Fuzzy time series, interval ratio, Alabama enrollment forecasting.

ABSTRACT

Background: *One forecasting method that is often used is time series forecasting. The development of applied mathematics has encouraged new mathematical findings that led to the birth of new branches of mathematics, one of which is fuzzy.*

Purpose: *The objectives of the study, namely forecasting, fuzzy set, time series, fuzzy time series, fuzzy time series Singh, interval ratio and measurement of accuracy level.*

Method: *This research method applies Chen's fuzzy time series in the section of determining the universe of talk you to the fuzzification of historical data and in the part of forecasting results obtained through a heuristic approach by building three forecasting rules, namely Rule 2.1, Rule 2.2, and Rule 2.3 to obtain better results and affect very small AFER values. As well as making modifications to the interval partition section using interval ratios to be able to reflect data variations.*

Results*: Based on the calculation of AFER values for order 2, order 3, and order 4 respectively obtained at 1.06389%, 0.689368%, and 0.711947%. Therefore, it can be said, Singh's fuzzy time series forecasting method based on the ratio of 3rd-order intervals is better than that of 2nd-order and 4th-order.*

Conclusion*: Based on the results of research and discussion that has been carried out, it can be concluded that Singh's fuzzy time series forecasting method has the same algorithm as fuzzy time series forecasting. Singh's fuzzy time series forecasting method based on interval ratios applies fuzzy time series and Singh forecasting. Singh's fuzzy time series forecasting modification accuracy rate based on interval ratios produces excellent forecasting values according to evaluator average forecasting error rate (AFER).*

INTRODUCTION

One forecasting method that is often used is time series forecasting. Time series forecasting based on values observed in the past is subsequently used to predict future data. The relationship between mathematical theory and real-world problems gave rise to the terms pure mathematics and applied mathematics. According to Bell, (2012) The development of applied mathematics has encouraged new mathematical discoveries that led to the birth of new branches of mathematics. One branch of mathematics that continues to grow is fuzzy (Bělohlávek, Dauben, & Klir, 2017).

Inaccuracies and incompleteness of past data stemming from a rapidly changing environment (Morice, Kennedy, Rayner, & Jones, 2012). In addition, the decisions made by experts are subjective and depend on the competence of each one. Therefore, it is more appropriate when the data is presented in fuzzy numbers rather than firm numbers. Fuzzy has a vague or vague meaning. The fuzzy set was first discovered by Zadeh in 1965. The use of fuzzy allows a problem formulation to be solved with an accurate solution. Along with the times, a new method emerged that combines fuzzy with time series analysis, namely fuzzy time series. The way fuzzy time series works is that crips data is converted first into a form of linguistic data commonly called fuzzy sets. The first fuzzy time series introduced by Song & Chissom (Song and Chissom, 1993a) is a method of data based on fuzzy principles. Establishing fuzzy relationships and fuzzification of time series is the top priority of a model to predict fuzzy time series.

The application of fuzzy time series to forecasting the number of University of Alabama registrants was first investigated by Song and Chissom in 1993 (Song & Chissom, 1993). That same year Song & Chissom, (1993), developed the FTS method into a time-variant fuzzy time series model using a 3-layer back propagation neural network to defuzzify and apply it to the University of Alabama enrollment dataset. Then Chen, (1996) proposed a method that is more efficient than Song & Chissom, (1993) which is to use simplified arithmetic operations also apply to the University of Alabama enrollment dataset. The same application was also made Singh, (2007) by proposing a better and more versatile forecasting method based on the FTS forecasting concept of developing a form of simple computational algorithm. Later that same year Singh generalized from previous research with the aim of making it a powerful forecasting method (S. R. Singh, 2007). The FTS application to forecast the number of University of Alabama registrants was also carried out by Chen, Zou, & Gunawan, (2019) with interval proportioning methods and particle swarm optimization (PSO) techniques.

Many applications of FTS forecasting in particular focus on interval partitioning. In 1996, Chen first conducted FTS forecasting research using the Average-Based length method to determine effective partitions (Chen, 1996). Then Huarng, (2001) found the distribution-based length interval partitioning method, and the results of his research were quite effective compared to the Average-Based length method discovered by (Chen, 1996). Furthermore, research on determining interval pertiation based on frequency density (Chen & Hsu, 2004; Jilani, Burney, & Ardil, 2007). Then in 2006, Kunhuang Huarng proposed a new method of determining the length of intervals based on ratios (Huarng & Yu, 2006). After that Chen et al., (2019), proposed a new fuzzy time series (FTS) forecasting method based on interval proportions and particle swarm optimization (PSO) techniques.

Singh's fuzzy time series research Singh, (2007) is a development and simplification of the fuzzy time series Chen, 1996; Song & Chissom, (1993) in forecasting University of Alabama enrollment. The development of Singh's fuzzy time series forecasting method lies in the forecasting part, which uses a simple computational algorithm using difference parameters as fuzzy relations. While simplifying Singh's fuzzy time series forecasting method, because it is able to minimize the complexity of calculating fuzzy relational equation matrices that use complex min-max composition operations and the time consumed by various defuzzification processes. However, the weakness or rather the part that can be explored to obtain better forecasting performance values from Singh's fuzzy time series research is the interval partition section that uses the 7 interval partition rule. Therefore, the author aims to propose Singh's fuzzy time series forecasting method based on interval partitioning that is more effective, namely interval ratio. The method proposed on the thesis was applied to the University of Alabama enrollment dataset. Next, measure the performance of forecasting results using Average Forecasting Error Rate (AFER).

RESEARCH METHODS

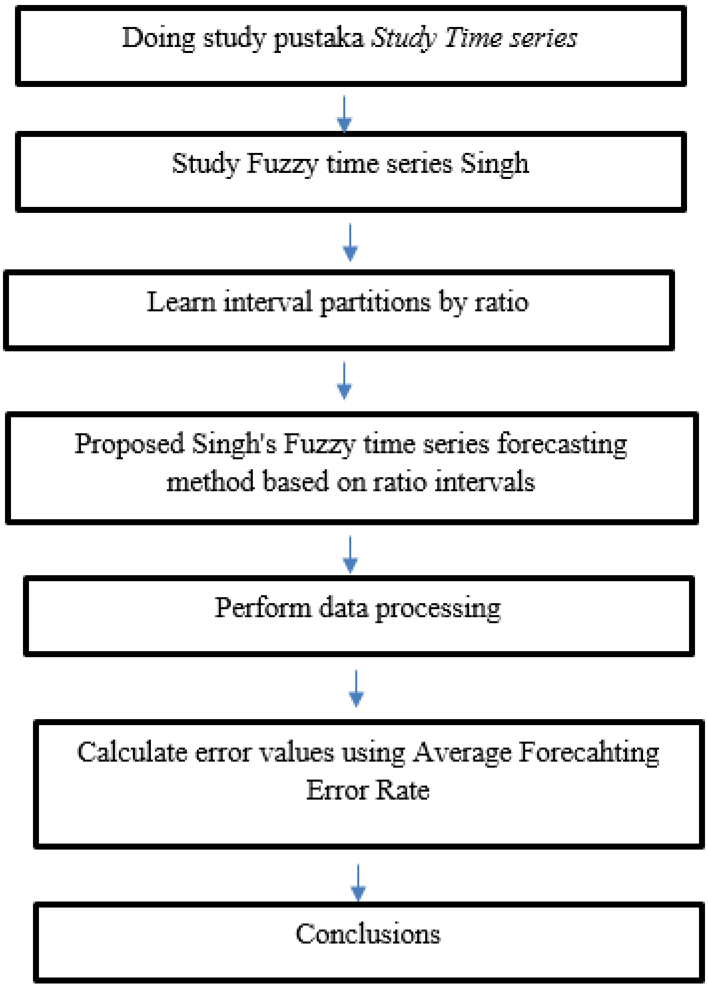

Figure 1. Research procedure flowchart

The data used in this study were secondary data obtained from journals (Song & Chissom, 1993). This data is data on the number of applicants for the University of Alabama. This study used data on the number of University of Alabama applicants from 1971 to 1992. By selecting data on the number of University of Alabama registrants, it is expected to provide more accurate estimates than forecasting (Chen, 1996; Chen & Hsu, 2004; Chen et al., 2019; Huarng, 2001; Huarng & Yu, 2006; Jilani et al., 2007; S. R. Singh, 2007; Song & Chissom, 1993).

The research method used in this study is a literature review, namely by collecting references in the form of books, journals and writings published on the website. From this method (Kumar & Gangwar, 2015), Singh's fuzzy time series forecasting algorithm can be determined based on interval ratio partitions to solve forecasting problems.

RESULTS AND DISCUSSION

After getting the forecasting results, the next step is to evaluate the forecasting results using *the average forecasting error rate* (AFER). The following is given an example of AFER calculation for order 3:

AFER=t=1n|At-Ft|Atn×100%

AFER=0,009275+0,008253+0,000331+…+0,00945519×100%

AFER=0,0068937×100%

AFER=0,68937%.

Detailed calculations *of average forecasting error rate* (AFER) for order 2, order 3, and order 4 are presented in Table 4.11, Table 4.12, and Table 4.13. The following table evaluates forecasting results using *the average forecasting error rate* (AFER) for order 2:

Table 1 Evaluation of forecasting results for order 2

Years	Total of registration	Forecasting Results	|At-Ft|	|At-Ft|At
1971	13055			
1972	13563			
1973	13867	13864,68	2,32	0,000167
1974	14696	14559,69	136,31	0,009275
1975	15460	15556,3	96,3	0,006229
1976	15311	15157,57	153,43	0,010021
1977	15603	15419,29	183,71	0,011774
1978	15861	15920,55	59,55	0,003754
1979	16807	16688,07	118,93	0,007076
1980	16919	17713,76	794,76	0,046974
1981	16388	16312,87	75,13	0,004584
1982	15433	15587,59	154,59	0,010017
1983	15497	15587,59	90,59	0,005846
1984	15145	15237,14	92,14	0,006084
1985	15163	15410,12	247,12	0,016298
1986	15984	15946,11	37,89	0,00237
1987	16859	16746,53	112,47	0,006671
1988	18150	17704,26	445,74	0,024559
1989	18970	19127,57	157,57	0,008306
1990	19328	19127,57	200,43	0,01037
1991	19337	18941,27	395,73	0,020465
1992	18876	18912,55	36,55	0,001936
Average forecasting error rate (AFER) %				1,06389%

The following table evaluates forecasting results using *the average forecasting error rate* (AFER) for order 3:

Table 2. Evaluation of forecasting results for order 3

Years	Total of registration	Forecasting Results	|At-Ft|	|At-Ft|At
1971	13055			
1972	13563			
1973	13867			
1974	14696	14559,69	136,31	0,009275
1975	15460	15587,59	127,59	0,008253
1976	15311	15316,07	5,07	0,000331
1977	15603	15603,05	0,05	0,000003
1978	15861	15946,11	85,11	0,005366
1979	16807	16688,07	118,93	0,007076
1980	16919	17111,45	192,45	0,011375
1981	16388	16312,87	75,13	0,004584
1982	15433	15587,59	154,59	0,010017
1983	15497	15616,3	119,3	0,007698
1984	15145	15237,14	92,14	0,006084
1985	15163	15263,07	100,07	0,0066
1986	15984	15946,11	37,89	0,00237
1987	16859	16737,53	121,47	0,007205
1988	18150	18277,15	127,15	0,007006
1989	18970	19127,57	157,57	0,008306
1990	19328	19166,54	161,46	0,008354
1991	19337	19112,29	224,71	0,011621
1992	18876	18697,53	178,47	0,009455
Average forecasting error rate (AFER) %				0,689368%

The following table evaluates forecasting results using *the average forecasting error rate* (AFER) for order 4:

Table 3. Evaluation of forecasting results for order 4

Years	Total of registration	Forecasting Results	|At-Ft|	|At-Ft|At
1971	13055			
1972	13563			
1973	13867			
1974	14696			
1975	15460	15587,59	127,59	0,008253
1976	15311	15201,21	109,79	0,007171
1977	15603	15587,59	15,41	0,000988
1978	15861	15929,8	68,8	0,004338
1979	16807	16688,07	118,93	0,007076
1980	16919	17093,3	174,3	0,010302
1981	16388	16342,93	45,07	0,00275
1982	15433	15587,59	154,59	0,010017
1983	15497	15640,53	143,53	0,009262
1984	15145	15237,14	92,14	0,006084
1985	15163	15237,14	74,14	0,00489
1986	15984	15946,11	37,89	0,00237
1987	16859	16688,07	170,93	0,010139
1988	18150	18277,15	127,15	0,007006
1989	18970	19127,57	157,57	0,008306

<table>
<tr><th>Years</th><th>Total of registration</th><th>Forecasting Results</th><th>|At-Ft|</th><th>|At-Ft|At</th></tr>
<tr><td>1990</td><td>19328</td><td>19127,57</td><td>200,43</td><td>0,01037</td></tr>
<tr><td>1991</td><td>19337</td><td>19127,57</td><td>209,43</td><td>0,010831</td></tr>
<tr><td>1992</td><td>18876</td><td>18725,01</td><td>150,99</td><td>0,007999</td></tr>
<tr><td colspan="4">Average forecasting error rate (AFER) %</td><td>0,711947%</td></tr>
</table>

Based on Table 1, Table 2, and Table 3, AFER for order 2, order 3, and order 4 respectively amounted to 1.063891%, 0.689368%, 0.711947%. Furthermore, based on Table 1, because the three AFER values are less than 10%, it can be concluded that the forecasting results have very good criteria. Then from the AFER results obtained, it can be concluded that Singh's *fuzzy time series* forecasting method based on the ratio of intervals to order 3 is the best method compared to order 2 and order 4.

A comparison chart visualization of Singh's *fuzzy time series* forecasting method based on the interval ratio between order 2, order 3, and order 4 can be seen in the following graph:

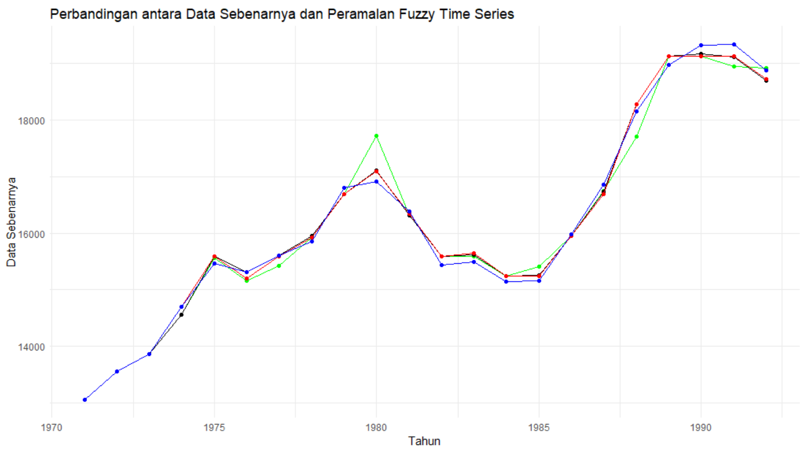

Figure 2. Comparative graph between actual data and order 2, order 3, and order 4

The blue graph shows the actual number of University of Alabama registrants, the green graph shows Singh's *fuzzy time series* forecasting results based on the 2nd-order interval ratio, the black graph shows Singh's *fuzzy time series* forecasting results based on the 3rd-order interval ratio, and the red graph shows the *fuzzy time series* forecasting resultsSingh based on the ratio of 4th-order intervals. From the graph above, it shows that the pattern of *Singh's fuzzy time series forecasting results* based on interval ratios is almost the same as the actual data on the number of University of Alabama registrants, although the resulting forecasting value is not the same as the actual data on the number of University of Alabama registrants.

Based on Figure 2, it can be seen that the green graph has a considerable error because the forecasting results are not close to the actual data, for example in 1980 and 1991. Then for the red chart is better than the green chart because the pattern is closer to the actual data. However, compared to the black color chart, for example, in 1976 and 1990, it is not good enough because the black color graph is closer to the actual data. From the description above, it can be concluded that Singh's *fuzzy time series* forecasting results based on the ratio of 3rd order intervals are closer to the actual data compared to order 2 and order 4. This is because there are differences in the formation of *fuzzy logical relationships* that affect forecasting results.

Singh's fuzzy time series *forecasting* based on interval ratios for Year 1993

Singh's fuzzy time series *forecasting method* based on interval ratios was used to forecast data on the number of University of Alabama enrollees in 1993. Based on Table 4.6, obtained *fuzzy logical relationship* order 2 in 1993 is A21,A20. Because *fuzzy logical relationship* (A21, A20→) is not the same as *fuzzy logical relationship* (AiAj) so that the forecasting results in 1993 for order 2 do not follow Rule 2.1, but in the form of average forecasting results corresponding to A21 and A20 namely 18941.27+18912.552=18926.91. As for the 3rd order *fuzzy logical relationship* in 1993 it was A21,A21,A20. Because *fuzzy logical relationship* (A21,A21,A20→) is not the same as *fuzzy logical relationship* (AiAj) so that the forecasting results in 1993 for order 3 do not follow Rule 2.2, but in the form of average forecasting results corresponding to A21,A21 and A20 namely 19166,54+19112,29+18697,533=18992,12. As well as for *fuzzy logical relationships* of order 4 in 1993 were A21,A21,A21,A20. Because *fuzzy logical relationship* (A21,A21,A21,A20→) is not the same as *fuzzy logical relationship* (AiAj) so that the forecasting results in 1993 for order 4 did not follow Rule 2.3, but in the form of average forecasting results corresponding to A21,A21,A21 and A20 namely 19127.57+19127.57+19127,57+18725,014=19026.93.

The method proposed by the researcher is Singh's *fuzzy time series* forecasting method based on interval ratios. Singh's *fuzzy time series* forecasting method based on interval ratios applies *fuzzy time series* (Chen, 1996) in the section of determining the speech universe you to fuzzification of historical data which includes *fuzzy logical raltionship* (FLR) and *fuzzy logical raltionship group* (FLRG). Starting from the determination of the universe of U-talk to the fuzzification of historical data which includes FLR and FLRG, there is one step that plays an important role in forecasting *fuzzy time series*, namely determining the interval partition that will affect FLR, forecasting results, and the level of forecasting accuracy.

Fuzzy time series *forecasting* (Chen, 1996) still uses classical interval partition determination by dividing the U-talk universe into equal-length intervals. The disadvantage of determining interval partitions of equal length is that it may not reflect data variations precisely because *time series* data have a tendency to fluctuate (up or down). Therefore, to correct weaknesses, the determination of interval partitions using interval ratios (Huarng &; Yu, 2006) with the aim of being able to reflect data variations and assist in forecasting *time series* data and determining intervals based on ratios is considered better than the same interval length.

The next problem that arises and becomes a concern after knowing that ratio is a suitable approach to determining interval length is how to determine the *percentile sample* ratio. First, if the *percentile sample* ratio is set too large then there will be no fluctuations in the fuzzy time series. Suppose the *percentile sample ratio* is determined 200% of the first interval of the time *series* data 5000, 5100, ..., 10000, 10100. Causes the first interval between 5000 and 15000 (5000+5000×200%). Indicates that the result is undesirable because it is unable to describe fluctuations. Second, if the *percentile sample ratio* is set too small, then the *fuzzy time series* becomes trivial, maybe even the same as the original *time series* data . Based on the same *time series* data , for example the ratio is too small, say 0.02%, the first interval. Causes the first interval between 5000 and 5001 (5000+5000×0.02%), the second interval between 5001 and 5002, and so on. From this case, it is possible that the *fuzzy time series* will be very close to the original *time series* data , which is also undesirable. Therefore, guidelines appear in determining the *percentile sample ratio* , which must be large enough so that the length of the interval will not be trivial. Intuitively the *percentile sample ratio* is set to 50%.

Furthermore, another step that plays an important role so that it affects forecasting results in addition to determining interval partitions is the process of forming *fuzzy logical relationship groups* (FLRG). In *fuzzy time series* forecasting (Chen, 1996), if the FLRG of Ai is (Ai→A1,A2,...,Am) then Ft+1=A1,A2,...,Am and the *crisp* value is the average of the *fuzzy* set corresponding to the middle value of the interval (Chen, 1996). However, Singh *'s* fuzzy time series forecasting method *based on interval ratios is different from (Chen, 1996) because the forecasting concept is based on* the upper *and lower and midpoint* bounds. Therefore, if there is a FLRG of Ai is (Ai→A1,A2,...,Am) then Ft+1=A1,A2,...,Am and determines the average of the *fuzzy* set corresponding to the upper and lower bounds and then obtained *the midpoint*. Thus, the forecasting result Ft+1 is obtained by forecasting according to Rule 2.1, Rule 2.2, or Rule 2.3. However, if there is an empty FLRG of Ai (Ai), then the forecasting result is Ft+1=Ai.

The forecasting results in Singh's *fuzzy time series* forecasting method based on interval ratios are obtained through *a heuristic* approach, in contrast to *fuzzy time series forecasting* (Singh, 2007a) which uses computational algorithms that aim to minimize the complexity of calculating min-max composition operations in the FLR equation and time in the defuzzification process. In simple terms, *heuristics* are simple guidelines or rules that are commonly used by humans in assessing something or used to make decisions. The construction of three forecasting rules, namely Rule 2.1, Rule 2.2, and Rule 2.3 aims to obtain better results and affect very small AFER values.

Rule 2.1 is used for *2nd order fuzzy time series* forecasting , Rule 2.2 is used for *3rd order fuzzy time series* forecasting , and Rule 2.3 is used for *4th order fuzzy time series* forecasting . The three rules have almost the same steps, which differ only in the initial step, namely the addition of a new difference parameter (Dt) (Yang & Shami, 2020). The establishment of new difference parameters as *fuzzy relations* is applied to the *current state* to estimate the value of the next state in order to better accommodate possible data vagueness and make it a powerful method. The new difference parameter for Rule 2.1 is Dt=|Xt-Xt-1|, Rule 2.2 i.e. Dt=|Xt-Xt-1|-|Xt-1-Xt-2|, and Rule 2.3 i.e. Dt=|Xt-Xt-1-|Xt-1-Xt-2-|Xt-2-Xt-3|.

The formation of a new difference parameter (Dt) results in the need to add four other parameters, namely Mt1=Nt1=Ot1=Xt+Dt, Mt2=Nt2=Ot2=Xt-Dt, Mt3=Nt3=Ot3=Xt+Dt/2, and Mt4=Nt4=Ot4=Xt-Dt/2 where Mtp is for Rule 2.1, Ntp is for Rule 2.2, and Otp is for Rule 2.3 with p∈1,2,3,4. The addition of more than one parameter produces many forecasting results so that it is expected to be able to reflect data variations through the average of the forecasting results obtained. In Singh's *fuzzy time series* forecasting method based on interval ratios, forecasting based on *upper* and *lower* limits is expected to have flexibility in approaching actual data (Pritpal Singh & Borah, 2013)v. . Then if the forecasting result of the four parameters is not in the closed interval of *the fuzzy* or unqualified set more than equal to the lower bound and less than equal to the upper limit, then the forecasting result (Ft+1) is the *midpoint* value of the interval (Pritpal Singh, 2017).

Meanwhile, implementing into the University of Alabama enrollment data because related *fuzzy time series* research has been done previously by (Song & Chissom, 1993b), (Chen, 1996), (Huarng, 2001), (Huarng &; Yu, 2006), (Singh, 2007a), (Singh, 2007b), (Jilani et al., 2007), and (Zou et al., 2019). And by doing forecasting on the same data, we can find out how effective Singh's *fuzzy time series* forecasting method is based on interval ratios than previous *fuzzy time series forecasting methods* by (Song &; Chissom, 1993b), (Chen, 1996), (Huarng, 2001), (Huarng &; Yu, 2006), (Singh, 2007a), (Singh, 2007b), (Jilani et al., 2007), and (Zou et al., 2019). The effectiveness of the forecasting method obtained can be determined based on the value of *the average forecasting error rate* (AFER). Research (Song & Chissom, 1993b) obtained AFER 3.22%, (Chen, 1996) obtained AFER 3.11%, (Huarng, 2001) obtained AFER 2.45%, (Huarng &; Yu, 2006) obtained AFER 0.73%, (Singh, 2007a) obtained AFER 1.53%, (Singh, 2007b) obtained AFER 1.46%, (Jilani et al., 2007) obtained AFER 1.02%, and (Zou et al., 2019) obtained (AFER) 0.73%.

The effectiveness of Singh's *fuzzy time series* forecasting method based on interval ratios can be known when *the average forecasting error rate* (AFER) value is very small. Based on the calculation of AFER values for Singh's *fuzzy time series* forecasting method based on interval ratios on order 2, order 3, and order 4 respectively are 1.06389%, 0.689368%, 0.711947%. Therefore, it can be said, Singh's *fuzzy time series* forecasting method based on the ratio of 3rd order intervals is better than order 2 and order 4 and better than *fuzzy time series* research by (Song & Chissom, 1993b), (Chen, 1996), (Huarng, 2001), (Huarng & Yu, 2006), (Singh, 2007a), (Singh, 2007b), (Jilani et al., 2007), and (Zou et al., 2019). Thus, Singh's *fuzzy time series forecasting method* based on effective interval ratios was used for forecasting the University of Alabama registrant number set data.

CONCLUSION

Based on the results of research and discussion that has been carried out, it can be concluded that Singh's fuzzy time series forecasting method has the same algorithm as fuzzy time series forecasting. Singh's fuzzy time series forecasting method based on interval ratios applies fuzzy time series and Singh forecasting. Singh's fuzzy time series forecasting modification accuracy rate based on interval ratios produces excellent forecasting values according to evaluators average forecasting error rate (AFER).

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